ARTICLE

Rethinking the optimal methods for vector analysis of astigmatism

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Purpose: To evaluate the accuracy and usefulness of certain methods of analyzing astigmatic vectors.

Setting: Cullen Eye Institute, Baylor College of Medicine, Houston, Texas.

Design: Case samples.

Methods: Using 2 sample cases for analysis of corneal surgically induced astigmatism and an actual toric intraocular lens (IOL) case, univariate analyses from the ASSORT program were compared to double-angle plots of preoperative and postoperative astigmatism and prediction errors.

Results: Certain univariate figures for analyzing the 2 sample cases were misleading. For the toric IOL case, some of the key outcome vectors were inaccurate.

Conclusions: ASSORT's univariate analysis of astigmatic vectors can be unpredictably erroneous and misleading. Recommended vector analyses should include double-angle plots with centroids and confidence ellipses of preoperative and postoperative astigmatism and the prediction errors, along with means and standard deviations of these vector magnitudes.

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The primary goals of astigmatic analysis are to answer 2 key questions: (1) how accurate was our treatment, and (2) what can we learn from our data to improve the outcomes. In our previous editorial, we discussed the merits of vector analysis of astigmatism using double-angle plots.[1] Previously, the editors of JCRS and JRS adopted a method of astigmatism analysis (ASSORT) that provides a number of univariate analyses of astigmatic vectors.[2,3] As we will discuss further, we believe that many of these graphs are flawed and should be abandoned.

Astigmatism is complicated. There are 2 categories of information that we want to know when we analyze astigmatic procedures: (1) certain scalar values: mean ± SD of preoperative and postoperative astigmatism, independent of angle, and percentages of eyes within certain ranges (±0.25 diopter [D], 0.5 D, etc.); these are the numbers that matter most to both patient and clinician; and (2) vector analysis: what changes did the surgery induce and how did these changes: differ from the intended target; these are the data that matter to the scientist/clinician in analyzing what actually occurred.

Reporting scalar outcomes of preoperative and postoperative magnitude of astigmatism is straightforward because they are single variables without direction or angle; they are, therefore, amenable to univariate analysis. The problem arises with vector analysis. Astigmatism has both magnitude and direction, but the latter is actually bidirectional because the axis or meridian is made up of 2 semimeridians. As a result, it does not satisfy the definition of a *Euclidean vector* without doubling the angle as determined by Stokes.[4] After the angle is doubled, it becomes a Euclidean vector because there is only 1 direction. The proper term is *double-angle astigmatism vector*. When this is completed, all vector algebra can be performed, after which the double angle is then halved for the single-angle result.[1]

The double-angle astigmatism vector can be broken down into 2 component eigenvectors with cosine and sine functions. As we will show further, depending on the vectors being analyzed and the type of analysis, univariate analysis of direction/angle alone, certain comparisons of vector magnitudes without angles, and using ratios of the magnitude alone can give incomplete and misleading results. Looking at the literature over the past many years, all methods of vector analysis—save one—analyze astigmatic outcomes by some version of bivariate analysis that includes doubling the angle and decomposing astigmatism into x and y components using sine and cosine functions, etc. Univariate vector analyses are restricted to means, standard deviations, standard errors,

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Table 1. Angular error (±degrees) corresponding to a magnitude error (D) at different measurement precisions.[8]

	Measured Magnitude (D)						
Tolerance (D)	0.125	0.250	0.50	1.00	2.00	4.00	8.00
±0.125	30	14	7	4	2	1	0
±0.25	90	30	14	7	4	2	1
±0.50		90	30	14	7	4	2
±1.00			90	30	14	7	4

The formula is θ = ArcSin [0.5 × (*t*/*M*)], where *t* is the tolerance (precision, eg, ±0.5 D), *M* is the magnitude of astigmatism, and θ is the angular error that is equivalent to the tolerance in diopters. Note that you need to know angle and magnitude.

confidence intervals, and *P* values for type I errors of the magnitudes of preoperative and postoperative vectors and of the vectors for prediction errors.

As we discussed previously, we believe that double-angle plots are better than single-angle plots at accurately portraying the essence of the data, attractive as the latter are because they match what we see in a phoropter. Double-angle plots allow us to optimally show the spread of the data clearly with accurate representation of their relative position to all the other points, and they allow depiction of the centroids, confidence ellipses, and the standard deviations of the astigmatic data. More importantly for the reader, once one grasps the concept of the double-angle plots, the data are visually easier and more accurate to interpret.

To the authors' knowledge, among publicly available programs for analyzing astigmatic outcomes, only the ASSORT program uses univariate analyses of vector magnitude and angle separately beyond those noted above. As we will show, some of these are inaccurate and misleading. Næser pointed out the potential errors in univariate analysis and stated: "However, refractive data are multivariate and cannot properly be described by univariate indices. The indices 'Magnitude of error' and 'Angle of error' might be appropriate for an individual patient but actually represent separate analyses of astigmatic direction and magnitude. Aggregate analysis will therefore yield systematic errors."[5–7] Table 1 from the article by Holladay et al. in 2001 shows the necessity of looking at angle and magnitude together for vector analysis.[8] The purpose of this article is to look more closely at these issues regarding potential flaws in certain univariate analyses of astigmatism.

METHODS

Graphical Display

To compare the ASSORT displays to the Astigmatism Double-Angle Plot tool that we developed, we used a dataset of eyes of patients undergoing cataract surgery.

Univariate Analysis of Corneal Surgically Induced Astigmatism (SIA)

To evaluate the ASSORT Corneal Analysis component of the ASSORT Group Analysis Calculator, we prepared 2 sample cases:

1. Case 1: preoperative 43 D/44 D @ 90 and postoperative 42.5 D/44.5 D @ 90 when the target is 43.5 D/43.5 D, that is, correction of 1 D at 90 degrees.
   - The SIA is 1.0 D @ 90 instead of at 180 degrees as planned.
   - Thus, the astigmatism became worse along the 90-degree meridian, not better.
2. Case 2: preoperative 43 D/44 D @ 90 and postoperative 42.5 D/44.5 D @ 135 again when the target is 43.5 D/43.5 D, that is, correction of 1 D at 90 degrees.
   - The SIA is 2.24 D @ 148 instead again of at 180 degrees as planned.
   - In this case, the astigmatism increased and along a different meridian.

Outcomes of Toric IOL Implantation

To evaluate the ASSORT Toric IOL Analysis tool in the Group Analysis Calculator, we used an actual straightforward clinical case:

1. Preoperative corneal astigmatism: 1.58 @ 96.
2. Abulafia–Koch Hill-RBF toric calculator predicts residual refractive astigmatism of 0.06 D @ 9 for 10.0 D SN6AT3 IOL (toricity 1.50 D at IOL plane) @ 99.
3. Final outcome: 0 D of refractive astigmatism.

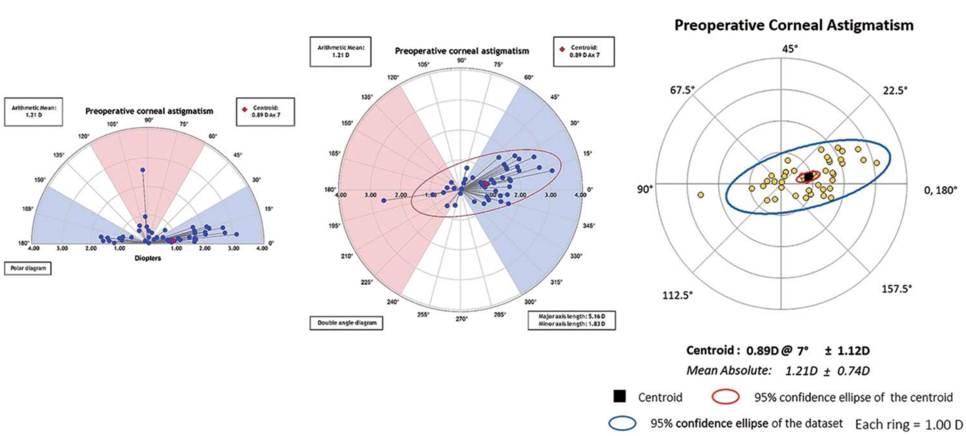

Figure 1. The same dataset displayed with the ASSORT polar (*left*) and double-angle (*middle*) plots and the Astigmatism Double-Angle Plot tool (*right*).

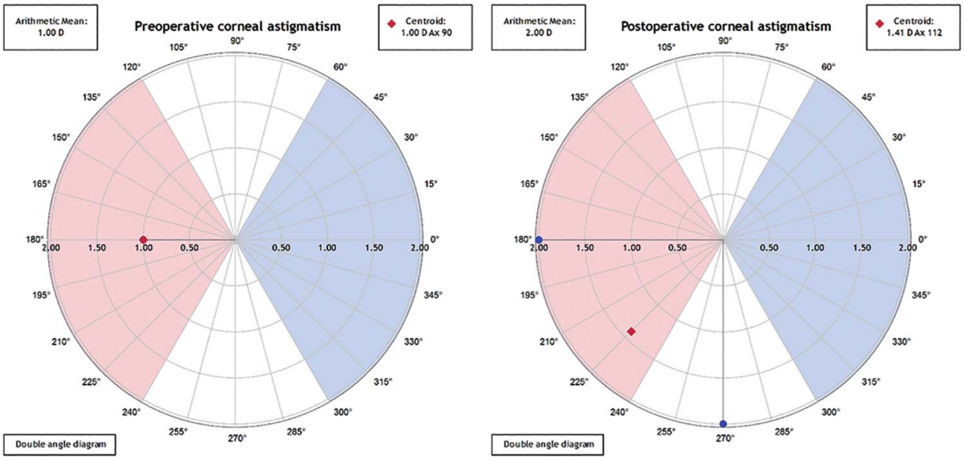

Figure 2. ASSORT double-angle plots of preoperative (*left*) and postoperative (*right*) astigmatism of the 2 sample corneal surgically induced astigmatism cases.

RESULTS

Graphical Display

Figure 1 shows the same dataset displayed with the ASSORT polar and double-angle plots and our Astigmatism Double-Angle Plot tool. The data trends are much more readily discerned with the double-angle plots. However, the ASSORT double-angle plots are labeled in a confusing manner: from 0 to 360 degrees instead of 0 to 180 degrees.

To illustrate this more clearly, Figure 2 is the ASSORT double-angle plot of preoperative and postoperative astigmatism of our 2 sample corneal SIA cases. It seems that the preoperative corneal astigmatism is at 180 degrees (instead of at 90 degrees) and that the postoperative astigmatism is at 180 degrees and 270 degrees (instead of at 90 and 135 degrees). Figure 3 is the ASSORT double-angle plot of the SIA. Because of the graph mislabeling, case 1 looks like that change was at 180 degrees (instead of at 90 degrees) and case 2 at 296 degrees (instead of at 148 degrees).

Corneal SIA

The flaws in certain univariate analysis of astigmatic vectors are demonstrated in the following 3 figures, using the data from our 2 sample cases with SIA:

1. Figure 4 is the ASSORT plot of SIA vs target-induced astigmatism (TIA). It does not accurately inform us of what occurred. For case 1, the datapoint is on the identity line, indicating no overcorrection or undercorrection, that is, the SIA and TIA are equal, implying accurate correction—instead of the actual 2 D of undercorrection in the wrong direction. The datapoint for case 2 correctly shows the more than 2 D SIA, but, without knowing the angle, it is unhelpful in describing what actually occurred.
2. Figure 5 is the angle of error plot. One datapoint is at 90 degrees, presumably case 1; it is unclear why it is not at −90 degrees because the treatment went in the opposite direction to the intended effect. The angle for the second case is at −35 to −25 degrees. Although correct,

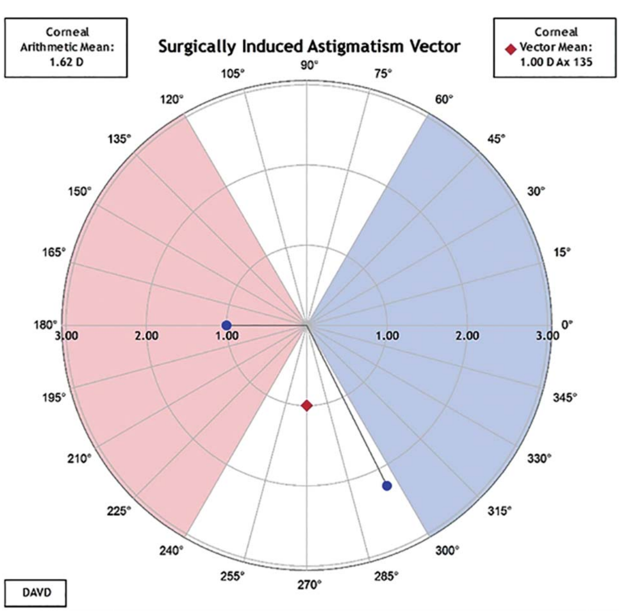

Figure 3. ASSORT double-angle plot of surgically induced astigmatism of the 2 sample cases.

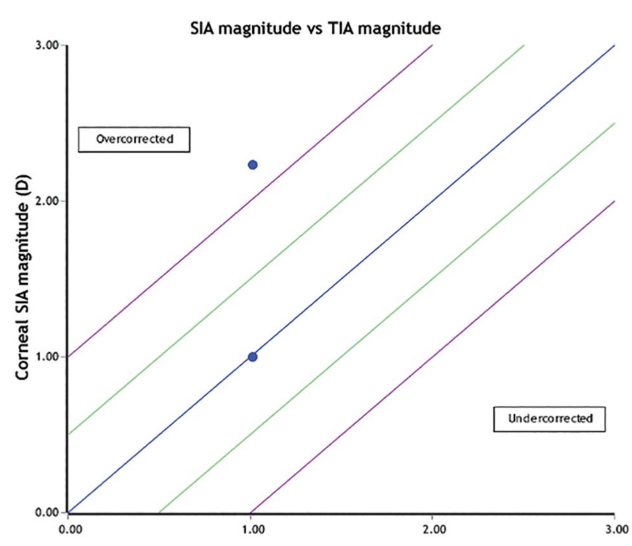

Figure 4. ASSORT plot of SIA vs TIA of the 2 sample cases (SI = surgically induced astigmatism; TIA = target-induced astigmatism).

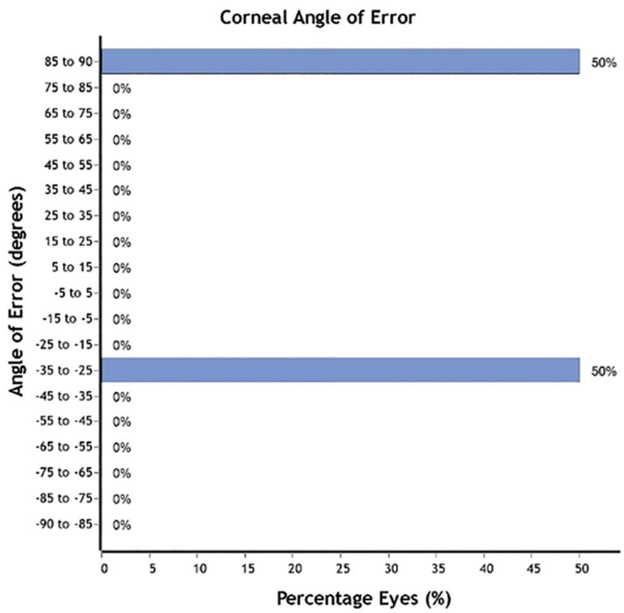

Figure 5. ASSORT angle of error plot of the 2 sample cases.

without knowing the vector magnitude for this angular error, this datum is not helpful. For example, one could envision angle of error plots of 5 cases within ±0.1 D of target but off by 60 degrees and 1 case with the correct angle but off by 2 D. The graph would suggest that the problem with the data is the angle of the correction, which clearly mischaracterizes the outcome.

3. Figure 6 is the polar plot of the correction index (CI). The definition of this is as follows: "Calculated by determining the ratio of the SIA to the TIA by dividing SIA by TIA. The CI is preferably 1.0. It is greater than 1.0 if an overcorrection occurs and less than 1.0 if there is an undercorrection."[9] The errors inherent in this should be obvious: a CI independent of angle is irrelevant at best. The data from our 2 cases make this point. On the ASSORT graph, there are 2 points, both along the 180-degree line, one at 1 and one at 2.2. Neither makes any sense. To correct the error for case 1, whose astigmatism goes in the wrong direction, an index is meaningless. For case 2, why would the datapoint be on the 180-degree line? There is no indication on this graph of the fact that the error for case 2 was at an angle that is oblique to the preoperative angle. Moreover, why is angle even included in the plots because the CI is by definition simply a ratio? The corneal geometric mean is likewise of no value in understanding the data. Nothing here helps us interpret what happened or how to improve the outcome.

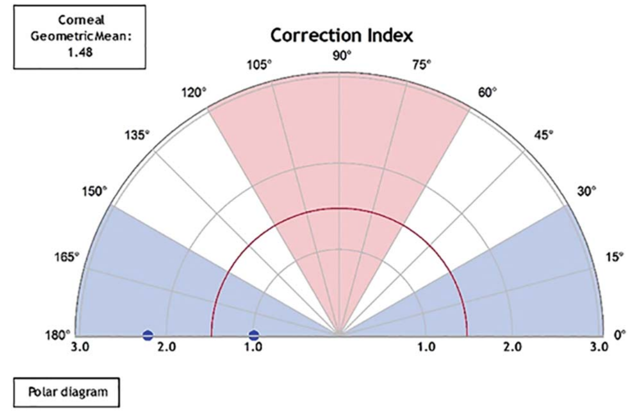

Figure 6. ASSORT polar plot of the correction index of the 2 sample cases.

Outcomes of Toric IOL Implantation

Analyzing our case with toric IOL with the ASSORT Toric IOL Analysis tool, we found the following flaws in the data analysis:

1. SIA: 1.58 @ 6 (Figure 7, A). Incorrect: ~1.02 D @ 99 was corrected, not 1.58 D.
2. The TIA in ASSORT is 1.02 @ 9 (Figure 7, B). This is the amount of astigmatism that was to be corrected by the toric IOL and is correct.
3. CI: 1.54 (Figure 8). This indicates that there is an overcorrection of 50%. This is obviously incorrect because the result was essentially perfect. Even if one inputs total corneal astigmatism of 1.00 to account for the posterior corneal astigmatism, the correction index is still 1.54, which would lead one to think that the astigmatism had been overcorrected.
4. Difference vector (DV): 0.57 @ 91 (Figure 9); with total corneal astigmatism used, the DV is still 0.57 D. This is incorrect because SIA and TIA should give a DV of near zero if the desired amount of astigmatism is fully corrected.

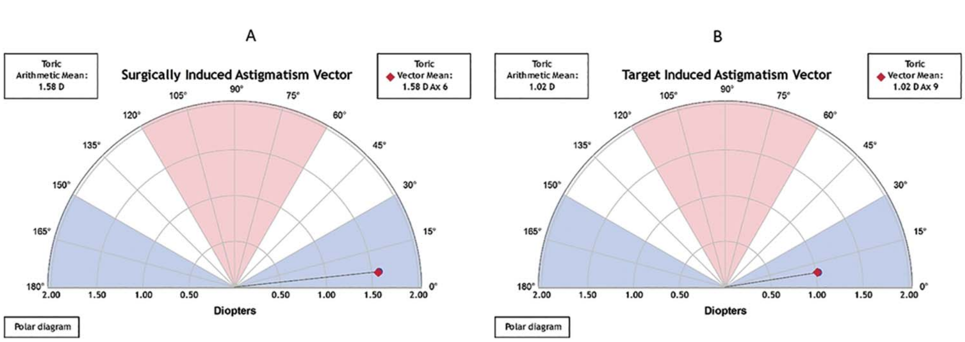

Figure 7. *A*: Surgically induced astigmatism of the toric IOL case plotted using the ASSORT Toric IOL Analysis tool. *B*: Target-induced astigmatism of the toric IOL case plotted using the ASSORT Toric IOL Analysis tool.

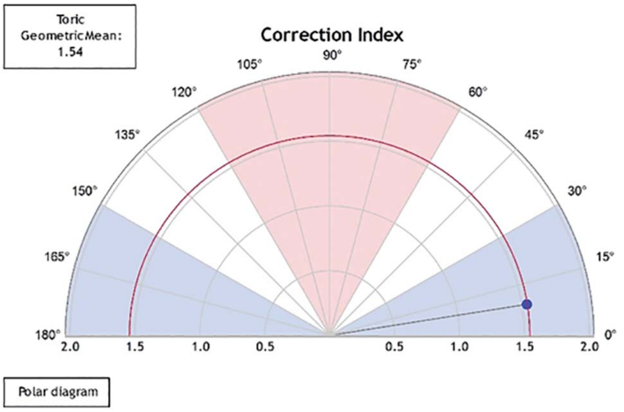

Figure 8. Correction index of the toric IOL case plotted using the ASSORT Toric IOL Analysis tool.

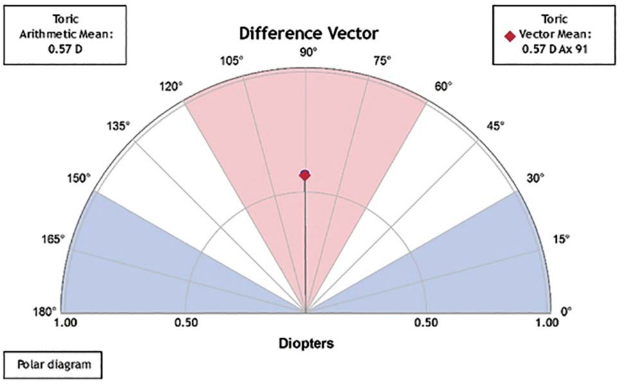

Figure 9. Difference vector of the toric IOL case plotted using the ASSORT Toric IOL Analysis tool.

5. Figure 10 is the correct plot with our Double-Angle Plot tool, showing a near-perfect prediction error for correction of the astigmatism.

Why these errors? ASSORT uses the preoperative keratometry as the baseline for the SIA calculation regardless of what is entered for total keratometry; this will automatically give incorrect values for SIA, DV, angle of error, and CI.

DISCUSSION

These examples demonstrate the inherent flaws in several of the univariate vector analyses of astigmatism provided in the ASSORT program. Univariate analysis works for certain scalar vector values: means and standard deviations of preoperative, postoperative, and prediction error vectors. It fails for the

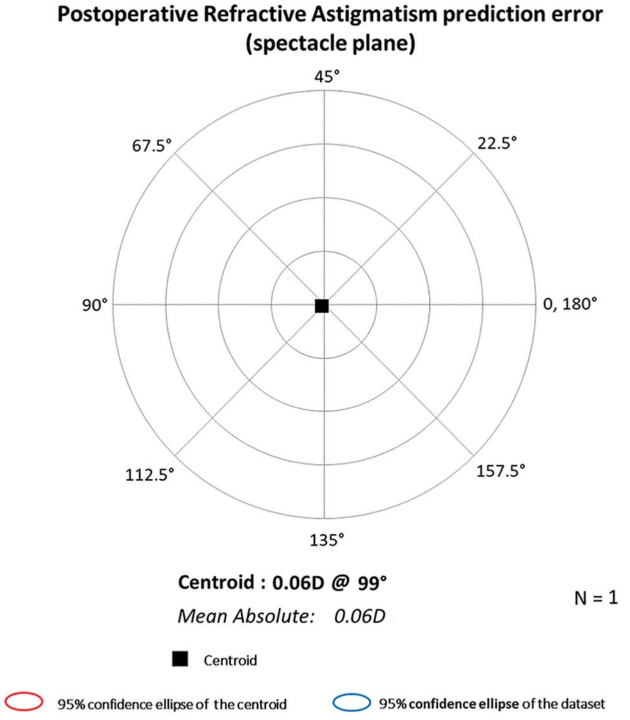

Figure 10. Refractive astigmatism prediction error of the toric IOL case plotted using the Astigmatism Double-Angle Plot tool.

ASSORT plots of TIA vs SIA, angle of error, and CI; the data that these graphs provide can be highly misleading and do not demonstrate what actually occurred. One could argue that our first 2 cases are rare or unusual and not representative of standard outcomes. However, that is exactly the point: looking at the ASSORT figures, one would never know that these cases are unusual. These graphs cannot be trusted to show the actual outcome errors.

What are the optimal figures for displaying astigmatic outcomes? We believe that those we provided in our editorial are the primary figures to include in astigmatic analysis. Vector displays should consist at a minimum of double-angle plots of preoperative and postoperative astigmatism and of the prediction error. The prediction error is precisely what is needed to demonstrate the deviation of the intervention from the desired correction. In addition, we need to know the prediction error to compare different formulas, for example, compare one toric calculator with another. The ASSORT program does provide the DV, which is the difference between the TIA and SIA. However, a graph of DV is not in the 9 figures recommend by the journals; also, as described above, DV is incorrectly calculated in the ASSORT Toric IOL Analysis tool, and the double-angle plots in ASSORT are incorrectly labeled.[3]

Reinstein et al. thoughtfully advocated in favor of single-angle plots for these reasons: "(1) single-angle polar plots do not require any further learning or understanding than what is taught to all ophthalmologists/optometrists, (2) data plotted on a single-angle plot are directly transferrable to the clinical situation of a topography, treatment, or eye, and (3) single-angle plots require less space on the page."[3] Our response would be that, although we agree with the first 2 points that there is an advantage to visualize polar plots compared with phoropters, topography, etc., (1) surely clinicians who can make it through medical or optometry school can be taught how to interpret double-angle plots, (2) eliminating invalid figures will save much more journal space than is consumed by switching to double-angle plots, (3) as noted earlier, only double-angle plots allow us to show the correct relationships with confidence ellipses and centroids for the data, and (4) double-angle plots make it much easier to compare outcomes of different procedures.

Finally, we want to share some thoughts about terminology. In our editorial in JCRS, we deliberately chose the term "prediction error" for astigmatism because that is the same term that is used for evaluating the accuracy of spherical equivalent intraocular lens power calculations.[1,10] It is, therefore, terminology that everyone should understand. Moreover, a term that has been coined in the Alpins method is "ocular residual astigmatism." This is a relatively new term that is vague and goes in the face of over 100 years of other terminology that is more accurately descriptive: "external astigmatism" (the anterior corneal surface) and "internal astigmatism" (all astigmatism from the corneal endothelium to the retina). Those 2 terms precisely capture the 2 components.

The errors inherent in univariate vector analyses in ASSORT were described by Næser.[5–7] As noted earlier, we found additional incorrect computations in the ASSORT program. We do not claim to have all of the answers regarding astigmatism analysis because the science in this area continues to evolve. As this occurs, our journals should adapt to advocate the most scientifically valid methods for astigmatism analysis.

WHAT WAS KNOWN

- Astigmatism has both magnitude and direction, and the analysis is complicated.
- Univariate analysis of astigmatism and bivariate analysis of astigmatism have been used.

WHAT THIS PAPER ADDS

- Certain univariate analyses of astigmatism are unpredictably erroneous and misleading.
- Double-angle plots provide a highly informative way to display astigmatic vectors.

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Disclosures: *Dr. Koch is a consultant for Alcon Laboratories, Inc., Carl Zeiss Meditec AG, and Johnson & Johnson Vision Care, Inc. Dr. Wang is a consultant for Carl Zeiss Meditec AG and Alcon Laboratories, Inc. Dr Abulafia is a consultant for Alcon Laboratories, Inc., Carl Zeiss Meditec AG, and Haag-Streit AG. Dr. Holladay is a consultant to Alcon Laboratories, Inc. and Carl Zeiss Meditec AG. The other author has a financial or proprietary interest in any material or method mentioned.*